A GOAL PROGRAMMING APPROACH TO CLASS C SCHOOL PLANNING

Robert Lee Prath

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A GOAL PROGRAMMING APPROACH TO CLASS C SCHOOL PLANNING

by

Robert Lee Prath

Thesis Advisor: C.R. Jones

March 1972

Approved for public release; distribution unlimited.

A Goal Programming Approach to Class C School Planning

by

Robert Lee Prath
Lieutenant Commander, United States Navy
B.S., United States Naval Academy, 1964

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1972

ABSTRACT

Enlisted personnel planners in the Bureau of Naval Personnel are tasked with meeting quantitative and qualitative requirements in the Navy's enlisted manpower force. In many cases fulfillment of qualitative requirements involves the allocation of personnel to training programs. Several analytical and computational models are proposed which allow planners to determine the levels at which various competing personnel requirements should be met. The levels prescribed for programs involving training may be used to formulate training plans. The models utilize quadratic programming techniques.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

This study originated from a suggestion by Mr. R.K. Lehto, Bureau of Naval Personnel, Washington, D.C.. The author is indebted to Mr. Lehto for assistance in collecting background information and in beginning the research effort. Deep appreciation is due Professor C.R. Jones of the Naval Postgraduate School for the friendly encouragement and stimulating counsel which he provided during preparation of the paper. Finally a thank you is due my wonderful wife for her warm encouragement during the years of graduate school.

I. INTRODUCTION

The phenomenon of concern in this paper is a training planning problem faced annually by enlisted personnel managers in the Bureau of Naval Personnel (BUPERS). BUPERS is a central administrative agency of the United States Navy whose objective is to:

> "....plan and direct the procurement, distribution, administration, career motivation, education, and training assigned, of Navy personnel, including those of the Naval Reserve, to meet the quantitative and qualitative manpower requirements of the United States Navy, as determined by the Chief of Naval Operations."[1]

Hence, an overall goal of BUPERS is to achieve an on-board personnel force which meets the specified parameters of a required force, in a maximal sense. In general, the parameters of the required personnel force are determined, exogenously to BUPERS, by the Chief of Naval Operations.

Within the Navy, enlisted personnel resources and enlisted personnel requirements are identified primarily, in terms of broad occupational skill categories, termed ratings. A rating defines an enlisted career field which requires similar aptitudes, knowledge, and training. Collectively, the individual ratings form the Navy enlisted rating structure, which is the fundamental administrative tool for the management of enlisted personnel resources.[2] In general, Navy enlisted personnel are advanced in grade, trained, and distributed by rating.

---

[1]Department of the Navy, Bureau of Naval Personnel Manual, NAVPERS 15791B, p. 00-1, October 1971.

[2]Naval Personnel Research and Development Laboratory, Report WRM 70-26, The Navy Rating Structure and the Self Renewing Occupational Field (SROF) Concept: An Analysis, by Linsert, H. and others, March 1970.

The Navy rating structure is supplemented by the Navy Enlisted Classification (NEC) Coding System, which provides more specific identification of technical skills within the scope of the ratings. An NEC is a four-digit identifier assigned to both enlisted personnel and enlisted billets. When assigned to personnel of a rating, it identifies special skills or training beyond that generally associated with the rating, itself. When assigned to a billet, an NEC identifies a special requirement within a rating. In many cases in order to become qualified to fulfill a billet with an NEC requirement, a man must attend a formal course of instruction.

Within the enlisted personnel structure, there are requirements for certain numbers of personnel in each Navy rating. Within the ratings, there are additional requirements for specified numbers of personnel with various NEC qualifications. In order to meet the NEC requirements within a rating, BUPERS enlisted personnel planners prepare NEC training plans, annually. These plans specify numbers of personnel which should receive training which qualifies them to fulfill NEC identified billets. A relatively straight-forward model, the NEC Training Resources (NEC-TR) Model, is utilized by training planners to determine the Class C School plan for each rating.[3] A description of this model is contained in Appendix A. Basically, it formulates a training plan for each rating utilizing a replacement table technique. A list of requirements and a description of the personnel inventories for a set of NEC programs belonging to a rating are input to the model. The model is then used to determine the required number of inputs to each NEC program which

[3]Since formal training in support of NEC requirements is performed at a category of schools termed Class C Schools, the plans are typically referred to as Class C School plans.

exactly satisfies the projected personnel shortages. Thus, within a rating program, the NEC-TR model prescribes the maximum level of NEC training which is consistent with planned NEC requirements.

From purely an NEC requirements point of view, the maximal training policy of the NEC-TR model seems near optimal. From the viewpoint of aggregate rating requirements, however, it is possible to take issue with the NEC-TR solution.

Over a planning period, the decision to input rating qualified personnel to NEC training programs allocates available rating man-years to non-productive accounts (training and travel time associated with training). When, for an individual rating, there is a general shortage of rating man-years in a planning period, a maximal NEC training policy continues to insure that the NEC requirements are fulfilled; but it further aggravates the total rating man-year shortage in the period. BUPERS is tasked with ensuring that current personnel inventories are compatible with both total requirements by rating and NEC requirements within rating. It seems reasonable, therefore, that an optimum NEC training policy from a rating planning point of view should explicitly consider both the requirements of individual NEC programs and the overall manning requirements of the rating.

In this paper computational and analytical models are presented which allow managers to consider, collectively, the decisions to meet overall rating requirements and NEC requirements within a rating. These models may be used to determine the explicit effect of personnel shortages on the decisions to meet NEC training requirements. Of the three models presented, two are single period planning problems while the third may be used to consider the NEC training planning problem over

five periods (fiscal years). In the proposed models, the concept of quadratic loss is utilized to simulate the goal-seeking behavior of BUPERS planners relative to a set of competing personnel requirements within a rating.

II. SUBOPTIMIZATION

A. GENERAL

An organization the size of the United States Navy employs an extremely wide variety of resources, many of which have, at most, an indirect relationship to its overall objectives. It's unrealistic to expect that, in this environment, detailed decisions involving all the Navy's resources could be efficiently made by a central staff. Consequently, the Navy's decision-making processes are broken into various levels, the wider ranging decisions being made by higher level activities, other decisions being delegated to lower levels. Only at the highest level can decision-makers optimize with respect to the objectives of the entire Navy. At other levels, decision-makers may attempt to optimize, but only with respect to sub-objectives which are non-identical to final Navy objectives. Generally speaking, decision-makers below the highest level are, necessarily, "suboptimizers".

It would be desirable to be able to propose analysis which solved the problem of allocating resources, to support NEC requirements, from a global or Navy-wide point of view. In the author's opinion, however, it's not very realistic to believe that a single analysis, or in practice a single central staff, could consider the myriad of resource allocation trade-offs necessary to establish truly optimal NEC requirements and training levels. Even if the appropriate benefits of individual training programs were known, information regarding the value of alternative uses of the training resources would still be required. As a result the total amount of information needed would be roughly equivalent to that required to make all Navy resource allocations at a central

level. In practice, then, the question of whether or not to suboptimize in analyzing the NEC training problem is not very interesting. The interesting question, and the question with which the reader might be concerned, is the degree of suboptimization finally chosen. The analysis in this paper chooses, primarily, a BUPERS viewpoint towards the NEC training planning problem. This is largely because the study is addressed to users at the BUPERS level.

B. PRINCIPLE IMPLICATIONS OF SUBOPTIMIZATION

Supoptimizing at the BUPERS level affects both the scope and context of the study. Resource allocation alternatives, non-relevant at the BUPERS decision-making level, are eliminated from consideration.[4] In addition, it's assumed that many decisions made at higher levels are optimal, including those concerned with NEC requirements and total rating requirements of the enlisted manpower force. Finally, goals and the criteria utilized to measure goal achievement are reflective of BUPERS objectives rather than overall Navy objectives.[5]

A direct implication of the above is that the analysis need not be explicitly influenced by many of the pure budget costs of training, such as the cost of operating training plants and supporting training travel. It's certainly possible that insufficient availability of training plant or travel funds could constrain the level of training in

---

[4]For instance, it is generally not considered the prerogative of BUPERS to determine that resources programmed to meet NEC training requirements should be instead allocated to construction of a new class of ships.

[5]BUPERS objectives are hopefully related to, but certainly not identical with, final Navy objectives.

support of NEC requirements.[6] However, it's presumed that NEC requirements determined at a higher level are optimal. Therefore, when resources are available it's, in general, an optimal policy to expend them to fulfill a vacant NEC requirement. Trade-offs concerned with alternative uses of the resources, so expended, are considered to have been addressed, at least indirectly, at higher levels in the decentralized decision-making hierarchy.

---

[6]In fact, the TPGP formulations will accomodate such constraints.

III. INTRODUCTION TO THE TRAINING PLAN GOAL PROGRAMMING MODELS

A. GENERAL

The analysis in this paper is carried out by constructing several analytical and computational models called Training Plan Goal Programming (TPGP) Models. A main hypothesis of the paper is that there exists interdependencies between the decisions to meet overall rating requirements and NEC requirements within rating. The TPGP Models are, therefore, formulations which allow these decisions to be considered collectively. Decisions produced by the TPGP Models may then be compared to those which might have been reached had the NEC programs and the rating program been considered separately. Since decisions regarding these programs are made at roughly collateral levels within BUPERS, the author believes that it is feasible to make them interdependent, if it is proven desirable to do so.

In order to use the TPGP formulations a goal must first be established for each of the competing personnel requirements in the NEC training problem. These goals are incorporated into the TPGP Models. The model objectives are to determine the final allocation of resources (rating man-years) which most closely meets all of the program goals and at the same time does not exceed any of the technological, policy, or resource constraints which might have been imposed on the problem. In addition, if the planner has subjective priorities for fulfilling individual goals, he can utilize them to directly influence the allocation of personnel resources to various programs.

For reasons previously stated, the objective functions, or distributional criteria, of the TPGP formulations are not sensitive to many of

the budget costs associated with individual training programs. These costs will influence the final allocation of personnel to programs only when they are included as binding constraints on the problem. For instance, the models will find solutions which economize on training travel funds only when a budget constraint on travel funds is introduced into the formulation which makes an unconstrained solution unfeasible. There are costs, however, to which the model objective functions are always sensitive. These costs can perhaps be best described by introducing the concept of opportunity cost.

Opportunity costs are the measurable benefits foregone by rejecting the next best alternative use of resources. In the TPGP Models the basic resource, rating man-years, is allocated so as to most closely meet a set of goals for the rating and NEC programs. The opportunity cost of allocating a man-year to a given program are the benefits that would have been achieved by allocating the man-year to the next best program. Generally speaking, the objective of the TPGP Models in terms of opportunity cost, is to define an allocation of personnel to programs, such that the opportunity cost of allocating the next increment of resource to any program is identical.

B. QUADRATIC LOSS AND THE TRAINING PLAN GOAL PROGRAMMING CRITERIA

According to Hitch [3] "The criterion for 'good' criteria in operations research is always consistency with a 'good' criterion at a higher level." Presuming the overall objectives of BUPERS as a good higher criterion to be emulated, the TPGP formulations attempt to meet all personnel program requirements "as closely as possible." Essential to this procedure is an ability to satisfactorily measure the relative "closeness" of competing resource allocations with respect to a set of

personnel goals. Clearly, the selection of an appropriate measure involves judgement to some degree. Various persons or groups of persons, with differing points of view, are apt to argue for different resource allocations in the face of identical personnel goals. The viewpoint of primary concern in the case of this analysis, however, is that of the agency, BUPERS. To capture this point of view by attempting to measure the collective desires of applicable managers at BUPERS, seemed to the author a somewhat unrealistic endeavor. Therefore the author invoked a recurrent thesis of contemporary choice theory. That is, that persons (and agencies) tend to reveal their preferences in their actions.

A criterion commonly termed "quadratic loss" is used by the TPGP models to measure the goodness with which a given allocation of personnel resources satisfied a set of competing personnel goals. Quadratic loss has been put to use in a wide variety of statistical and managerial applications (see Ref. 4, Ref. 5 and Ref. 6, for instance).

Appendix B contains a simple example which motivates the use of the quadratic loss criterion in a personnel distribution problem. It can be seen from this example that a distribution of resources, under a quadratic loss criterion, is characterized by a basic propensity to "fair-share"[7] the resources to the programs. That is, where priorities for various programs are equivalent, and there are no constraints concerning the manner in which individual programs must be allocated to, the "quadratic loss" manager will maintain a fair-share of the total resources in each of the competing programs.

---

[7]In this paper a fair-share distribution is one in which shortages are shared by programs on an equal percent basis.

The concept of quadratic loss is utilized in the TPGP Models because it seems to provide an excellent simulation, from a historical point of view, of the unconstrained goal-seeking behavior of BUPERS, as an organization, relative to a set of competing personnel goals. This view has been confirmed by informal liaison with managers at BUPERS, the author's own experience, and planning directives and methods in existence at BUPERS. Reference 7, for instance, states:

> "The need for manning level priorities is dictated by the fact that shortages exist in many categories of personnel. Available personnel are distributed equitably, sharing the shortages that exist, except in special cases."

In a literal sense, application of quadratic loss to a personnel planning problem infers that a planner's dissatisfaction is proportional to the squared shortage of a program. It also infers that the programs have diminishing marginal returns. That is, one unit of resource supplied to a program manned at 60% produces greater return than one unit of resources supplied to a program manned at 95%. Whether a poll of planners at BUPERS would substantiate the above properties is actually not important, however. The important fact is that quadratic loss seems to provide a means for determining allocations of personnel which correlate well with those traditionally preferred by BUPERS' personnel managers. The exact reasons for this correlation are aside from the purposes of this study.

C. FORMULATION OF THE OBJECTIVE FUNCTION

1. The Personnel Program Goals

Since the TPGP models are goal programming models it was necessary to define an appropriate single goal for each of the two categories of programs which compete for resources in the NEC planning problem.

The single goal for the rating program over a planning period is that the average man-year shortage over the period be equal to zero. This shortage may be computed using a number of, essentially equivalent, formulas. A simple one is:

$$\text{(3.1)} \quad \begin{array}{l}\text{average man-year}\\ \text{shortage/excess}\\ \text{period } i\end{array} = \frac{\begin{array}{l}\text{shortage/excess}\\ \text{begin}\\ \text{period } i\end{array} - \begin{array}{l}\text{shortage/excess}\\ \text{end}\\ \text{period } i\end{array}}{2}$$

The goal form of (3.1) is equivalent to the goal "actual rating man-year average equals required rating man-year average" during period i.

Formula (3.1) assumes that the instantaneous shortage of men in a specific rating during a period (nominally, a fiscal year) can be approximated by a straight line. This assumption is used extensively in Navy personnel modeling. The author knows of no study which either supports or condemns this assumption; nor of any simple test which can be applied, such as variance of actual shortage from "straight line" shortage, to test its goodness.

The single goal for each NEC program over a planning period is obtained exogenously from the BUPERS NEC Training Resource (NEC-TR) Model outlined in Ref. 8. These goals are of the form "required school input per NEC program per planning period." Although the individual NEC program goals are adjusted for factors such as school attrition, they essentially define the required school input such that, for each NEC program, end-of-the-period personnel inventory equals end-of-the-year adjusted requirements. A description of the NEC-TR model and a discussion of why these goals were considered to represent the proper goals relative to the entire NEC planning period is contained in Appendix A.

The goal established for a rating program is measurable with respect to the entire planning period. Both the goal and deviations from it are measured in terms of man-years. The goals established for an individual NEC program are measured at a point during the planning period. Attempts to define NEC program goals in terms of NEC man-years were not successful; due to eccentricities of the training problem, discussed in Appendix A, these attempts invariably yielded models which compensated shortages in one portion of a planning period with excesses in another.

2. A Sample Objective Function

Goal formulations and concepts of quadratic loss discussed in previous sections can be combined with notation listed in Table I to produce a sample form of the TPGP Model objective function for one planning period. It is:

$$(3.2)\quad \text{Minimize}\quad \frac{(S_i + N_1 t_1 + \dots + N_m t_m)^2}{R_r} + \sum_{k=1}^{m} P_k \frac{(S_k - N_k)^2}{R_k}$$

In (3.2) there are m decision variables, N_k (k=1,...,m), representing student inputs to m NEC programs. Deviation from the rating goal is represented by the first portion of (3.2). It is the sum of the zero-training man-year shortage plus the additional man-year shortage resulting from decisions to train. Deviations from the single NEC program goals are represented as differences between the planned student input and the required input for each of the m programs.

Programs in formulation (3.2) are normalized for size. Each program is, therefore, divided by its respective personnel requirements.

TABLE I

Symbology for the single-period Training Plan Goal Programming Models.

PARAMETER	DESCRIPTION	UNITS	SOURCE	COMMENTS
R_r	Rating requirements over planning period	man-years	requirements plan	
S_i	Zero-training man-year shortage	man-years	ADSTAP	
R_k	Adjusted end requirements in the kth NEC program	men	NEC-TR Model	See Appendix A
S_k	Required school input to meet goal for the kth NEC program	men	NEC-TR Model	See Appendix A
t_k	Non-productive down time associated with training in the kth NEC program	years	historical data	
N_k	Number of men input to NEC training in support of the goal for the kth NEC program	men	decision varible	
P_k	Subjective priority of the kth NEC program			See Appendix C

This essentially provides a means to measure deviations from goals as a per cent of program size. Other properties of (3.2) are discussed in Appendix B.

Formulation (3.2) behaves in a manner which can be reasonably related to the NEC planning problem. The decision variables are in terms directly applicable to the planning problem. A decision to input an additional man to the k^{th} NEC program, (increase N_k by one unit), results in the loss of t_k man-years to the source rating. After training, the man "returns to the personnel force" qualified to fulfill both an NEC and a rating requirement.

The constant, t_k, in this paper is generally considered to represent down time due to training associated with the k^{th} NEC program. It can, however, be adjusted to amortize man-years lost to training staff. If, for instance, it is ascertained that five training staff man-years, from the source rating, are required for every twenty-five student inputs to the k^{th} program, then t_k would be appropriately inflated by two tenths of a man-year.

Objective function (3.2) is also adaptable to individual programs involving cross training. Suppose, for instance, that in a given problem formulation the i^{th} and the j^{th} program involve cross training. That is, a man must be qualified in the j^{th} NEC in order to be a candidate for the i^{th} NEC program. Then the i^{th} and the j^{th} program can be made interdependent by modifying (3.2) to:

(3.3) Minimize
$$\frac{(S_i + N_1 t_1 + \ldots + N_m t_m)^2}{R} + \ldots + \frac{P_1(S_i - N_i)^2}{R_i} + \ldots + \frac{P_j(S_j + N_j - N_j)^2}{R_j} + \ldots .$$

A note of caution accompanies this modification however. If cross training is accounted for utilizing one of the TPGP models, then factors compensating for cross training in the NEC-TR model should be eliminated.

D. FORMULATION OF THE CONSTRAINTS

All expected constraints on the TPGP objective function can be categorized as either resource, policy, or technological constraints. The ability of the TPGP models to incorporate these constraints into the training planning problem, itself, gives it a possible advantage over the replacement table solutions currently generated by the NEC-TR model. Training plans generated by the latter model must be negotiated during the planning cycle in order to make them feasible. Solutions to the training planning problem produced by the TPGP models are feasible with respect to all included planning constraints. They have the additional quality that, within assumptions of the model, they are the best of all the feasible solutions.

Since the TPGP models are intended to be solved by known quadratic programming techniques, it is mandatory that all constraints be formed as linear combinations of the TPGP decision variables (student input per NEC program). It is not considered practical to mention more than a few of the possible constraints on the NEC planning problem which can be handled in the TPGP format. By way of example, three hypothetical constraints are formulated.

The first example is a resource constraint due to a budget category. Consider that budget allocated to the sample rating for travel in support of Class C School training is B dollars. Of all programs in the problem, four of them require these funds in the amount C_1, C_2, C_3,

and C_4, respectively, for each student input. The resource constraint can be formulated as:

$$(3.4) \quad C_1N_1 + C_2N_2 + C_3N_3 + C_4N_4 \leq B .$$

Consider next a technological constraint due to training plant capacity. A school can adjust its curriculum so as to provide a total of 25 man-years of technical training of the type required by three individual NEC programs in a planning problem. The course lengths are CL_1, CL_2, and CL_3. An appropriate restraint expression is:

$$(3.5) \quad CL_1N_1 + CL_2N_2 + CL_3N_3 \leq 25 .$$

A possible policy constraint in a training planning environment might be that a certain NEC program is to be manned at a minimum of 90%. In this case an appropriate constraint formulation is:

$$(3.6) \qquad \frac{S_1}{R_1} - \frac{N_1}{R_1} \leq 0.1 ,$$

which implies that the shortage scheduled for NEC program one must be less than or equal to ten per cent in the TPGP solution.

Aside from the above examples an additional constraint is imposed on TPGP formulations. It is:

$$(3.7) \qquad N_k \geq 0$$

for all NEC programs. Condition (3.7) prevents training plan solutions which call for "untraining." It is a binding constraint to NEC programs which are projected to have excesses throughout a planning period.

IV. TWO SINGLE-PERIOD PLANNING MODELS

A. DISCUSSION

The predicament of one attempting to allocate resources to programs with competing goals is aptly described in Ref. 9 by Charnes and Stedry who state:

> "It may be tautological, but none the less interesting, that it is impossible to simultaneously optimize two functions; one can, at best, optimize one placing a constraint on the other, or one can construct a super-functional which is some function (perhaps a weighted sum) of the initial functions."

The first proposed Training Plan Goal Programming Model incorporates all program goals into the objective function. It, therefore, utilizes the latter of the two basic options which the above authors claim available to the analyst. By contrast, the second proposed TPGP model, which places a constraint on the deviation from the rating goal, uses a form of the remaining alternative.

B. MODEL I: A SINGLE-PERIOD POINT SOLUTION MODEL

1. Training Plan Goal Programming

Model I is designed to provide a point solution to the NEC planning problem for each Navy rating, over a single planning period. The objective of this model is to obtain an allocation of personnel which minimizes the squared deviations from the weighted program goals, thus meeting all goals "as closely as possible." The programs are assigned subjective weights proportional to their relative priority (see Appendix C). Higher weights are associated with the most critical goals. The remainder of the model structure is concerned with the various constraints which must be satisfied while trying to obtain an

allocation of rating man-years which minimizes squared deviations from the goals. A mathematical statement of the model, using symbology, listed in Table I, yields the quadratic program:

$$\text{(4.1)} \quad \text{Minimize} \quad \frac{(S_i + N_1 t_1 + \ldots + N_m t_m)^2}{R_k} \quad + \quad \sum_{k=1}^{m} P_k \frac{(S_k - N_k)^2}{R_k}$$

Subject to:

Resource Constraints
Technological Constraints
Policy Constraints
$N_k \geq 0 \quad k=1,\ldots,m$ NEC programs.

2. <u>Conditions For Unconstrained Optimality</u>

Letting S_r^o represent the man-year shortage in the rating program at optimality, and N_k^o represent the optimal personnel input to the k^{th} NEC program one obtains:

$$\text{(4.2)} \quad S_r^o = S_i + N_1^o t_1 + \ldots + N_m^o t_m \ .$$

Likewise, letting S_k^o represent the optimal man shortage in the k^{th} NEC program yields:

$$\text{(4.3)} \quad S_k^0 = S_k - N_k^o \quad k=1,\ldots,m \text{ NEC programs.}$$

Using equations (4.2) and (4.3) and simple optimization techniques one can obtain a statement of the relationships between various programs of Model I, at optimality, when none of the constraints are binding.[8] The relationship between the rating program and the k^{th} NEC

---

[8]Since objective function (4.1) is a convex function, the 1^{st} order conditions are necessary and sufficient for a global minimum (see Ref. 10).

program is:

$$(4.4) \qquad \frac{S_k^{\,o}}{R_k} = t_k \left(\frac{P_r}{P_k}\right) \left(\frac{S_r^{\,o}}{R_k}\right)$$

for k=1,...,m NEC programs. The relationship between the i^{th} and the j^{th} NEC program is:

$$(4.5) \qquad \frac{S_i^{\,o}}{R_i} = \left(\frac{P_j}{P_i}\right) \left(\frac{t_i}{t_j}\right) \left(\frac{S_j^{\,o}}{R_j}\right)$$

Condition (4.4) states that at unconstrained optimality, the percent man-shortage allocated to the k^{th} NEC program is proportional to the training down time associated with the k^{th} NEC program, the relative weight of the k^{th} NEC goal to the rating goal, and the percent man-year shortage in the source rating. Condition (4.5) states that the relative propensity to meet the requirements of the i^{th} and j^{th} NEC programs, is dependent on the ratio of their priorities and on the ratio of their training times.

3. Contrasts With the NEC-TR Model

In contrast to condition (4.4), the unconstrained optimality conditions of the NEC-TR model currently utilized by BUPERS are:

$$\frac{S_k^{\,o}}{R_k} = 0 \qquad k=1,\ldots,m \text{ NEC programs.}$$

From (4.4) it can be observed that when rating man-years are scarce ($S_r^{\,o} > 0$), and when a positive amount of training time is associated with an input to the k^{th} NEC program ($t_k > 0$), the two models will

not yield identical results, unless the relative priority of the rating man-year goal is zero. Since, in the author's opinion, it's difficult to envision a case in which rating man-years have no value, a zero priority for the rating goal is considered unrealistic. Therefore, TPGP Model I, which is sensitive to the personnel cost of training, can be expected to yield solutions to the NEC planning problem which differ from those of the NEC-TR model. When there is a shortage of rating man-years over a period, TPGP Model I will prescribe a training strategy in which the requirements of NEC programs are met at less than the one hundred per cent level, prescribed by the NEC-TR Model. The Model I per cent shortfall for an individual program is proportional to the training down time of the program and the expected man-year shortage in the source rating. It is inversely proportional to the subjective priority of the program. When there is an expected excess of man-years in a given rating over a planning period, excess training is prescribed for NEC programs which use that rating as a source. Again, the excess training is proportional to training down time and the per cent man-year excess. It is inversely proportional to the goal priority of a NEC program.

4. Effects Due to Training Down Time

Optimality condition (4.5) provides a mathematical statement of the effect of training down time on the Model I solution. It can be observed from this condition that, if all NEC programs in a planning problem were accorded equal priority, then the final shortages in these programs would still differ if program down times were different. In the case of equal priority, NEC programs with larger training times are allocated proportionately larger personnel shortages.

5. Unconstrained Optimality Conditions for Programs Involving Cross Training

Previous sections demonstrated a technique for incorporating cross training within NEC programs into an objective function of the type used in program (4.1). In this specific case of Model I this modification is of the form:

$$(4.9)\quad \text{Minimize}\quad \frac{P_r(S_i + N_1t_1 +\ldots+ N_mt_m)^2}{R_r} +\ldots.$$

$$+ \frac{P_i(S_i - N_i)^2}{R_i} + \frac{P_j(S_j - N_j + N_i)^2}{R_j} +\ldots.$$

where the i^{th} NEC program receives its inputs from the j^{th} NEC program. By direct differentiation of (4.9) it can be shown that at optimality:

$$(4.10)\qquad \frac{S_i^{o}}{R_i} = (t_i + t_j) \left(\frac{P_r}{P_i}\right) \left(\frac{S_r^{o}}{R_r}\right)$$

Thus, when the i^{th} NEC program draws on the j^{th} NEC program for its inputs, the effective down time is considered to be the total of the training down time associated with the i^{th} and the j^{th} program. This seems logical, since a decision to put a man into the i^{th} program indirectly infers a necessity to conduct additional replacement training in the j^{th} program.

6. Computational Experience

In order to visibly demonstrate Model I solution characteristics described in preceding sections, FORTRAN codes were developed for the IBM-360, and several sample problems were run. The main objective, in

design of the codes, was attainment of reliable solutions to the unconstrained planning problem, rather than efficiency or operational convenience.

Tables II and III contain Model I results in the case of two sample ratings. NEC-TR model solutions to the sample problems are contained in the rows marked "Prog. Shortage." These may be compared directly with the TPGP solutions. In an intuitive sense, it can be seen that the TPGP solution "trims" the NEC-TR solution. This "trimming" is proportional to the training time associated with an NEC program and the scarcity of rating man-years. It is inversely proportional to the goal priority of an NEC program.

Since TPGP Model I incorporates the resource of prime concern, rating man-years, into the objective function it has been permissible to direct most of the discussion concerning problem solution toward the unconstrained case. The unconstrained case was considered to be of special interest since current BUPERS planning methodology is to determine an "optimum" NEC training plan and then to make it feasible by negotiating the binding constraints with appropriate agencies.

In the case of Model I, of course, known NEC planning constraints may be imposed directly on the planning problem for a rating. The model will then determine the best allocation of personnel to training which meets the constraints. When a number of binding constraints are imposed on the Model I formulation, the only practical way to obtain a solution is to utilize a computer code designed to solve quadratic programs. There are many such codes in existence. An excellent one, described in Ref. 11, is available at the Rand Corporation, Santa Monica, California.

TABLE II. Model I, sample computational results for rating A.

	RATING	NEC PROGRAM								
		1	2	3	4	5	6	7	8	9
Total Requirements	5,000	50	100	300	500	200	300	50	300	300
Prog. Weight (P_k)	1.0	1.1	1.6	1.1	0.5	2.3	1.8	2.5	3.5	1.6
Down Time (t_k)		.70	.50	.54	.20	.20	.24	.50	.81	.79
Prog. Shortage (S_k)	S_i=700	18	33	73	125	35	75	25	29	60
Model I Input (N_k)		13	28	49	92	30	69	24	25	36

TABLE III. Model I, sample computational results for rating B.

	RATING	NEC PROGRAM								
		1	2	3	4	5	6	7	8	9
Total requirements	20,000	25	50	100	100	100	100	100	1000	2000
Prog. Weight (P_k)	1.0	2.0	1.8	1.6	1.4	1.4	1.7	1.7	1.9	1.55
Down Time (t_k)		.50	.50	.40	.40	.30	.30	.20	.50	.25
Prog. Shortage (S_k)	S_i=1200	12	20	60	60	60	40	40	500	1000
Model I Input		11	19	58	57	58	38	39	476	971

C. MODEL II: A MULTIPLE SOLUTION, SINGLE-PERIOD MODEL

1. Why Another Single-Period Model?

By incorporating the trade-off between NEC program goals and rating program goals into an analytical decision model, TPGP Model I establishes relationships between various NEC program planning parameters which might not be in strict consonance with the preferences and judgements of individual decision-makers. An example is Model I optimality condition (4.13) which prescribes a training decision,

$$(4.13)\qquad \frac{S_k^{\,o}}{R_k} = t_k \left(\frac{P_r}{P_k}\right)\left(\frac{S_r^{\,o}}{R_r}\right) ,$$

in which the shortfall from the k^{th} NEC program $\left(\frac{S_k^{\,o}}{R_k}\right)$ goal is proportional to the scarcity of rating man-years $\left(\frac{S_r^{\,o}}{R_r}\right)$. Literally, as the opportunity cost of allocating rating man-years to NEC programs increases, the tendency to expend them in support of NEC training decreases. This logic, however intuitive, may not appeal to all NEC training planners.

For instance, an individual planner, faced with increasing shortages of available rating man-years, may prefer to sacrifice an increased amount of the remaining man-years to training in order to fulfill NEC program goals. In other words, he might conclude that an increase in the percent man-year shortage in a rating generates a proportionately greater obligation that the remaining cadre of rating personnel meet or exceed specified requirements for NEC skills. It's possible that such preferences could be incorporated into optimality condition (4.13) of Model I. To accomplish this the P_k's, or NEC program

goal weights, would become increasing functions of the rating man-year shortage; the greater the rating man-year shortage, the higher the priority of the NEC program goals. However, actual measurement of the way certain NEC goal priorities might vary with respect to a change in rating shortage, for an individual planner, is by all appearances a complex, time-consuming problem. Model II circumvents this problem. Moreover, it removes from the model all relationships implicit in condition (4.13) and permits the training planner to make explicit judgements concerning the number of rating man-years he is willing to allocate to training. The model produces a set of NEC training plans which represent the "best" plans available for a given number of rating man-years expended in NEC training.

2. Training Plan Goal Programming Model II

The second proposed TPGP model incorporates only NEC program goals into the model objective function. Since all the goals in the objective function of Model II are of the same type, it is the author's opinion, that the assumption of quadratic loss, goal-seeking, behavior is all the more applicable in the case of this model. In consonance with the quadratic loss assumption, the objective of Model II is to minimize the squared deviations from the weighted NEC program goals. Fulfillment of this objective is explicitly limited by a constraint on the number rating of man-years allocated to NEC training. Model II, then, determines the allocation of available rating man-years, which most closely fulfills all NEC program goals and at the same time meets all planning constraints.

The mathematical formulation for Model II, using symbology previously defined, is the quadratic program:

(4.14) Minimize $\sum_{k=1}^{m} \frac{P_k(S_k - N_k)^2}{R_k}$ k=1,...,m NEC programs

(4.15) subject to $N_1t_1 + \ldots + N_mt_m \leq T$

resource constraints
policy constraints
technological constraints
$N_k \geq 0$.

As in the case of Model I, S_k^o , the optimal man shortage in the k^{th} NEC program can be written as:

(4.16) $S_k^o = S_k - N_k^o$ k=1,...,m NEC programs

In (4.16) N_k^o represents the optimal personnel input to the k^{th} NEC program. By setting the Lagrangian function formed by (4.14) and (4.15) above equal to zero one can obtain the relationships between the various NEC programs in an optimal plan for which only constraint (4.15) is binding. In this case the relationship between the i^{th} and the j^{th} program is:

(4.17) $\frac{S_i^o}{R_i} = \left(\frac{P_j}{P_i}\right)\left(\frac{t_i}{t_j}\right)\left(\frac{S_j}{R_j}\right)$ for all i,j = 1,...,m.

Condition (4.17) is identical to condition (4.5) of Model I. This implies that, at optimality, relationships between the NEC programs are identical for both Model I and Model II. If, for a given planning problem, the number of rating man-years allocated to training by a Model I solution were substituted for T in Model II then the actual solutions prescribed

by both models would be identical. As shown in Figure 1, the Model I solution is one of the solutions generated by Model II as the constraint on (4.15) is increased.

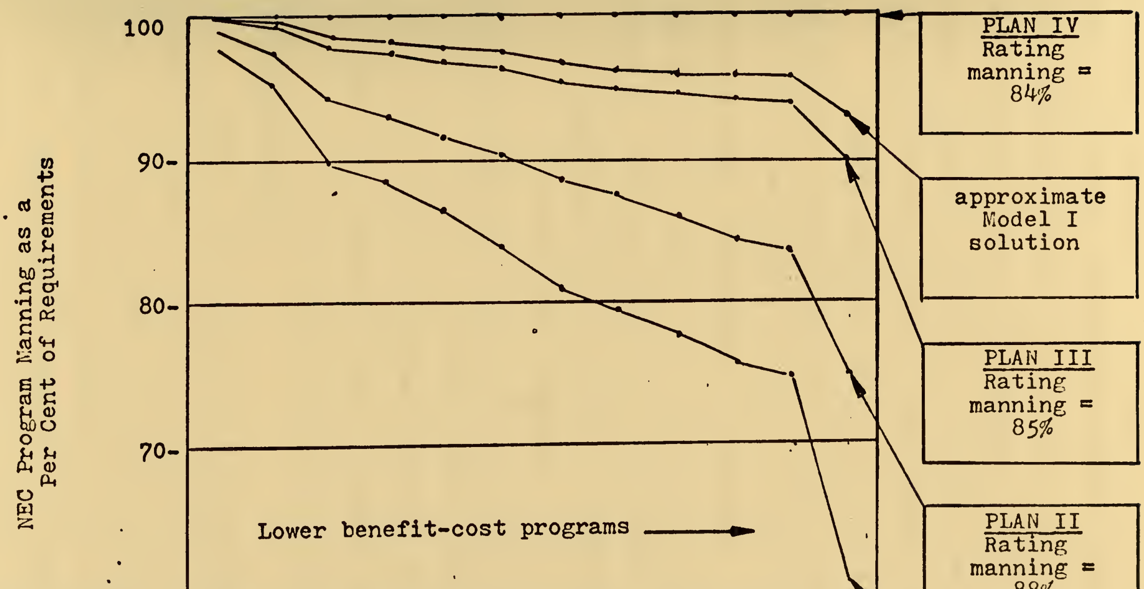
3. Incorporating Model II Into a Planning Strategy

Constraint (4.15) describes a restriction on, T, the number of rating man-years allocated to NEC training. For a specified value of T, Model II can be utilized to determine the best final manning postures for a collection of NEC programs relative to the goals for the programs. An explicit value of T also results in a certain manning posture for the source rating. This can be seen from the relation:

$$(4.17)\quad \frac{\begin{bmatrix}\text{man-years}\\ \text{available}\\ \text{period } i\end{bmatrix}}{\begin{bmatrix}\text{man-years}\\ \text{required}\\ \text{period } i\end{bmatrix}} = \frac{\begin{bmatrix}\text{man-years available}\\ \text{with zero NEC training}\\ \text{period } i\end{bmatrix} - T}{\begin{bmatrix}\text{man-years required period } i\end{bmatrix}}$$

Thus, as the parameter, T, is varied in Model II a planning frontier is mapped out which defines the best distribtuion of personnel resources to NEC programs over various levels of rating manning. Figure 1 is an example of a planning frontier which might result from such a process.

The weights for the sample NEC programs of Figure 1 are contained in the row labeled "P_k". Similarly, the associated training times are located in the row labeled "t_k". If P_k is considered a proxy for NEC program cost, then the benefit-cost ratio P_k/t_k can be defined for each of the programs. In Figure 1 the programs are ordered using this ratio, primarily for user viewing convenience. In addition, close review of the program ordering which results from assignment of benefit-cost ratios provides a means to recheck the validity of program weight



NEC Program	1	2	3	4	5	6	7	8	9	10	11	12
P_k	1.5	2.1	1.8	1.6	1.0	1.5	2.0	1.3	1.5	1.4	3.2	0.7
t_k	.11	.10	.24	.22	.17	.29	.48	.33	.42	.42	.37	.35
$\frac{P_k}{t_k}$	19.5	15	7.5	7.3	5.9	5.2	4.2	3.9	3.6	3.3	3.2	2.0

Figure 1. Model II planning frontiers for a sample problem.

assignments (see Appendix C). For the sample NEC programs in Figure 1, Model II was run for four specified values of T. Each run produced a "best" NEC manning plan which could be associated with a specific level of rating manning. The plans are graphed as Plans I through IV in Figure 1. Plan IV is essentially equivalent to the solution prescribed by the NEC-TR Model currently utilized by BUPERS planners.

In an NEC planning strategy which incorporated Model II, it's foreseen that a graphical means such as Figure 1 would be utilized to assist the training planner and/or rating manager in determining a trade-off between NEC manning and rating manning. The decision-maker need not limit his choices to the plans actually graphed, but could choose by interpolating between the plans presented.

4. Computing Solutions to the Model II Formulation

As is the case of all TPGP Models, Model II is a quadratic programming formulation. Again the most practical way to obtain numerical solutions is to utilize a high-speed computer code designed, especially, for solution of quadratic programming problems. The relatively simplistic nature of the objective function of Model II, makes it particularly easy to cast it into a format directly solvable by most quadratic programming packages.

Both the objective function and the constraints of Model II possess the property of separability. Reference 10 discusses, in detail, special computational methods which may be utilized to solve mathematical programming problems involving separable function. Many of these methods have been adapted for use in conjunction with high-speed computers. The IBM Mathematical Programming System/360 (MPS/360) is an example of an existing computer code which is capable of solving nonlinear programming

problems involving separable functions.[9] Informal liaison with BUPERS has revealed that the MPS/360 is available for use in the BUPERS IBM/360 installation.

D. USE OF MODELS TO PREDICT ACTUAL TRAINING DEMAND

Throughout this paper it has been convenient to take the position that Class C School Training Plans direct the allocation of personnel resources to specific training programs. Actually, rather than direct, the term "guide" more aptly describes the current function of these plans. In an economic sense, the Class C School Training Plans "prescribe" Navy-wide demand for specific training programs over a planning period.

Agencies responsible for the reprogramming of training assets to meet training requirements are most interested in estimating the actual training demand that will exist in a given program over a planning period. In the case of many Class C Schools, computation of real demand is based primarily on the demand prescribed in the various Class C School Training Plans. According to Ref. 13, prescribed training demand has traditionally exceeded actual training demand in many of the training programs. In view of this, an adjustment factor based on historical data is traditionally applied to training prescribed in a Class C School Plan in order to estimate real training demands.

Reference 13 cites personnel deficiencies as the primary reason for the difference between prescribed and actual demand for a training program. It's conceivable, in the author's opinion, that relative priorities of training programs is also a contributing factor. Models I and II NEC training decisions are sensitive to both of these factors,

---

[9]For a description of MPS/360, see Ref. 12.

(scarcity of rating man-years and program priorities). Therefore, if Model I and II Class C School training solutions are utilized to predict actual training demands for a program, historical adjustment factors should be used with some discretion.

V. A MULTI-PERIOD MODEL

A. CLASS C SCHOOL PLANNING IN THE LONG RUN

The final product of the Class C School planning process is a set of plans, one per rating, specifying school inputs by fiscal year for the five forthcoming fiscal years. This five year planning horizon is compatible with the Five Year Defense Plan (FYDP) established by the Department of Defense (DoD) in 1961 [14].

In the Class C School planning process, there are varying degrees of uncertainty associated with the values of most of the parameters.[10] As might be expected greater uncertainty is associated with the values of the planning parameters for the more distant fiscal years. As a result of these planning uncertainties, each training plan is reworked and revised, annually, for the following five fiscal years. Under this procedure (called a rolling schedule by Wagner [15]), a new fiscal year enters the horizon of each subsequent Class C School Plan as a past fiscal year is phased out. Because of variance in the planning parameters, a planned school input for an "out" fiscal year is apt to be altered several times prior to implementation as a current year input. Therefore, it can normally be expected that only the first year of a given Class C School plan represents a final plan.[11]

---

[10]A possible exception is training down time for an established NEC program.

[11]This is not to infer that the plan for, "out" years should be fixed. Like the FYDP, these plans do not represent hard-core commitments. They are tabulations based on programs tentatively approved by DoD. (Ref. 13, p. 1).

Since it's probable that a percentage of the students trained in one planning period will survive to fulfill NEC program requirements in future periods, there are inter-period dependencies, particularly between adjacent or nearly adjacent periods, in the NEC training planning problem. Additionally, the shortage of man-years within a rating is not constant, but normally varies from year to year, causing fluctuations in the opportunity cost of training from one period to the next.

The long run characteristics of the Class C School planning problem, addressed above, seem to suggest that a model which purports to establish an optimal training policy must possess a long run or multi-period planning capability. Since both TPGP Models I and II are single-period planning models, they quite probably prescribe training plans which are nonoptimal in the long run. In order to determine the diseconomies imposed on the planning problem by adopting a single-period solution, TPGP Model III was constructed. Model III is a multi-period extension of Model I.

B. MODEL III PLANNING PARAMETERS

1. General

Notation utilized in TPGP Model III is defined in Table IV. With exception of the symbology for continuation rates, notation is in general, identifiable with that utilized in the single-period models.

2. Model III Goal Weightings

The subjective priorities for the program goals in the case of TPGP Model III reflect more than a relative weighting of the programs within a planning period. They represent, in addition, the time preferences of the personnel planners. If planners have positive time preference then the goal weightings for the programs over time will

TABLE IV. Model III notation.

Symbol	Description	Units
S_{iy}	Zero-training man-year shortage for the rating program in the yth year	man-years
R_{ry}	Rating program requirements in the yth year	man-years
S_{ky}	Required man input to the kth NEC program during the yth year	men
R_{ky}	Adjusted end requirements of the kth NEC program at the end of the yth year	men
t_k	Non-productive down time associated with input to the kth NEC program	years
N_{ky}	Number of inputs to the kth NEC program during the yth year	men
P_{ry}	Subjective weight of the rating goal in the yth year	———
P_{ky}	Subjective weight of the goal for the kth NEC program in the yth year	———
C_{ij}^{k}	Continuation rate: probability that an input to the kth NEC in year i survives to year j	———

reflect the fact that goal fulfillment in earlier time periods is more preferred. By contrast, planners with negative time preference would exhibit a greater weighting for goal fulfillment in later time periods. Valid weighting, of the program goals for Model III, demands that the time preferences of personnel planners be properly accounted for.

In order to actually adjust program goals weights for time preference in a planning problem one might proceed in a manner which paralleled the following:

(1) Within each planning period, determine a relative weighting for all program goals belonging to the period. A procedure similar to one utilized for the single period model may be applied to each planning period (see Appendix C).

(2) Determine a time preference for the rating program goals over all planning periods.

Sample Method: Suppose, for example, a planning problem with five periods. The occupational rating program is chosen as the standard of value over time. Next one assumes an appropriate resource constraint over time. For the sample problem (Table V) it was assumed that there were sufficient rating man-years within the five year horizon to man the rating program at exactly 85% during each year. One may then establish a time preference by shifting the limited man-year resources between time periods until a most preferred allocation of resources to periods is obtained. A preferred allocation of resources to years, was assumed for the sample problem, and is listed in Table V. Having established a preferred allocation of resources over time, Table I.C in Appendix C, can be utilized to determine the weightings which this allocation of resources reveals. A description of the use of Table I.C

is contained in Appendix C. In this case, the values obtained from the table reflect subjective time priorities rather than relative weightings between the program goals within a year.

TABLE V. Determination of planner time preference; data for a sample case

Period	Initial allocation	Preferred allocation	Revealed weighting $\frac{P_{ry}}{P_{r1}}$	Present value factor
year 1	85%	92%	1.0	1.00
year 2	85%	89%	.77	.77
year 3	85%	86%	.60	.60
year 4	85%	82%	.45	.45
year 5	85%	76%	.35	.35

(3) Once a time preference for the rating goal has been established it may be utilized as standard of value to readjust the weightings of all program goals for time preference.

In Table V the last column of sample data are present value factors. Let:

$(PVF)_y$ = present value factor of the goal for the rating program in y^{th} period

P_{ry} = value of fulfilling the rating program goal in the y^{th} period

d_r = psychological discount rate for the rating goal.

The present value factor for a goal within a planning period is then defined by the relationship:

$$(5.1) \quad (PVF)_y = \frac{P_{ry}}{P_{ri}} = \frac{1}{(1 + d_r)^{y-1}} \qquad y=1,\ldots,Y \text{ time periods}$$

Equation (5.1) indicates that the present value factor is merely a ratio of the rating program goal weights which have been adjusted for time preference. Associated with each present value factor is a psychological discount rate, d_r. It provides another means of expressing the manner in which planner discounts goal-fulfillment benefits over time. For the sample data of Table V the psychological discount rate is not constant from one period to the next. However, (5.1) can be utilized to show that a constant psychological discount rate of about 30% provides a very good estimate of the present value factors, in the case of the sample data.

In the author's opinion, present value factors for the Class C School planning problem will be less than one, or conversely, discount rates for goal fulfillment will be positive. Having observed the planning process and conducted liaison with the planners the author contends that planners will normally prefer goal fulfillment in earlier periods to goal fulfillment in later periods. Uncertainty concerning the value of the planning parameters for future periods stimulates, further, the belief in positive psychological discount rates.

3. Continuation Rates

In order to numerically describe the way student inputs in one year survive to meet the NEC program requirements of future years, the concpt of a continuation rate is introduced. A continuation rate vector

is a series of probabilistic statements about the way individuals in a given personnel program survive from one year to the next. Normally the continuation rates for an enlisted personnel program are not constant. It can generally be expected, for instance, that in any Navy enlisted personnel program, the continuation rate for personnel going from their first to second year of service will differ significantly from the continuation rate of personnel going from their fourth to fifth year of service.

Continuation rates are routinely used tools in Naval enlisted manpower management. Continuation rate vectors are computed for Navy enlisted occupational ratings. To the author's knowledge, however, these vectors are not currently available for individual NEC programs.

Continuation rates utilized in Model III are of two forms. These are:

(1) $C^k_{y,y+1}$ = the probability that an individual in y^{th} year of service in NEC program k survives his $y+1^{st}$ year of service in NEC program k

(2) $C^k_{1,y}$ = the probability that an individual input to the k^{th} NEC program will survive his y^{th} year of service in the k^{th} NEC program.

C. A TWO-PERIOD MODEL

1. The Model

Prior to introduction of a generalized multi-period model, a two-period model is presented. This model is a simple one period extension of TPGP Model I. The two-period model presumes that all programs have been assigned weights proportional to their relative priority within period and that a present value factor has been applied to the goal weights of the second planning period. The objective of the

two-period model, then, is to determine the set of plans (one plan per period) which minimizes the present value of the squared deviations from the weighted personnel program goals. The remainder of the model formulation is concerned with the various constraints which must be satisfied in each planning period while trying to obtain the preferred allocation of personnel resources. A mathematical statement of the two-period model, using notation listed in Table IV, yields the quadratic program:

$$\text{(5.3)} \quad \text{Minimize} \quad \sum_{y=1}^{2} \frac{P_{ry}(S_i+N_{1y}t_1 + \ldots + N_{my}t_m)^2}{R_{ry}}$$

$$+ \sum_{k=1}^{m} \frac{P_{k1}(S_{k1}-N_{k1})^2}{R_{k1}} + \sum_{k=1}^{m} \frac{P_{k2}(S_{k2}-N_{k2}+C_{12}^{k}N_{k1})^2}{R_{k2}}$$

Subject to:

Resource constraints	$k=1,\ldots,m$ NEC programs
Technological constraints	$y=1,2$ years
Policy constraints	
$N_{ky} \geq 0$.	

2. Conditions for Unconstrained Optimality

The objective function of (5.3) may be differentiated with respect to the N_{ky}'s in order to obtain the necessary relationships between the various personnel programs at unconstrained optimality.[12] Performing this differentiation, one obtains the relations:

---

[12]Again, since the objective function of (5.3) is convex the first order conditions are necessary and sufficient for a global minimum.

$$(5.4) \qquad \frac{S^{o}_{k1}}{R_{k1}} = t_k \left(\frac{P_{r1}}{P_{k1}}\right)\left(\frac{S^{o}_{r1}}{R_{r1}}\right) - t_k C^{k}_{12} \left(\frac{P_{r2}}{P_{k1}}\right)\left(\frac{S^{o}_{r2}}{R_{r2}}\right)$$

$k=1,\ldots,m$ NEC programs.

The first term on the right hand side of (5.4) describes necessary conditions for optimality in the single period case. It follows, then, that the second term on the right hand side of (5.4) represents the effect of including a second planning period on the decision to meet first period NEC program goals. If the continuation rates, C^{k}_{12}, are zero, then solutions for both the one and two period models are identical. The same is true in the case that the rating man-year shortage is zero in the second period. If there is an excess of rating man-years in the second period, or

$$\frac{S_{r2}}{R_{r2}} \leq 0$$

then the propensity to meet NEC program goals during the first year is decreased. By contrast, a shortage of rating man-years in the second planning period increases the propensity to meet NEC program goals during the first year of the plan.

From (5.2) one can show that

$$P_{r2} = (PVF)_1 \times P_{r1}$$

Using the above relationship (5.4) can be rewritten

$$(5.5) \qquad \frac{S^{o}_{k1}}{R_{k1}} = t_k \left(\frac{P_{r1}}{P_{k1}}\right)\left(\frac{S^{o}_{r1}}{R_{r1}}\right) - C^{k}_{12}(PVF)_1 \left(\frac{S^{o}_{r2}}{R_{r1}}\right)$$

$k=1,\ldots,m$ NEC programs.

Using (5.5), the sensitivity of the single-period planning model to inclusion of an additional planning period can be determined for various values of the planning parameters. Based on brief exposure to historical continuation data it's the author's opinion that values for the continuation rates, C_{12}^{k}, will normally lie in range .80-.95. Also, it is estimated that $0 < (PVF)_2 < 1$. For cases in which these estimations are valid and there are mild shortages in the ratings for both periods, the principle effect of including a second planning period in the TPGP Model formulation will be to increase slightly the degree to which the first period training goals are fulfilled.

D. MODEL III: A FIVE-PERIOD PLANNING MODEL

1. General

The third proposed TPGP model has a five-period planning horizon. This particular planning horizon was selected because the author believed it satisfactory from both a theoretical and practical point of view. Theoretically, the five-period formulation seemed to define the pattern of interaction between the planning parameters and decision variables of various time periods, as the horizon was extended. Practically, there seemed to be no requirement for a model with a more distant planning horizon.

2. The Model

A mathematical statement of Model III is contained in Figure 2. As previously discussed, the goal weights, P_{ky}, are discounted in consonance with planner time preference. The objective of Model III is to determine a set of training plans for the five planning periods which minimizes the present value of the squared deviations from the weighted program goals. The plans are subject, of course, to all planning constraints within the five-year horizon.

$$(5.6) \quad \text{Minimize} \quad D = \sum_{y=1}^{5} \frac{P_{ry}(S_{iy}+N_{1y}t_1+ \ldots +N_{my}t_m)^2}{R_{ry}} + \sum_{k=1}^{m} \frac{P_{k1}(S_{k1}-N_{k1})^2}{R_{k1}}$$

$$+ \sum_{k=1}^{m} \frac{P_{k2}(S_{k2}-N_{k2}-C_{12}^{k}N_{k1})^2}{R_{k2}} + \sum_{k=1}^{m} \frac{P_{k3}(S_{k3}-N_{k3}-C_{13}^{k}N_{k1}-C_{12}^{k}N_{k2})^2}{R_{k3}}$$

$$+ \sum_{k=1}^{m} \frac{P_{k4}(S_{k4}-N_{k4}-C_{14}^{k}N_{k1}-C_{13}^{k}N_{k2}-C_{12}^{k}N_{k3})^2}{R_{k4}} + \sum_{k=1}^{m} \frac{P_{k5}(S_{k5}-N_{k5}+C_{15}^{k}N_{k1}-C_{14}^{k}N_{k2}-C_{13}^{k}N_{k3}-C_{12}^{k}N_{k4})^2}{R_{k5}}$$

Subject To: Resource Constraints

Technological Constraints

Policy Constraints

$N_{ky} \geq 0$

$k = 1,\ldots,m$ NEC programs

$y = 1,\ldots,5$ years

Figure 2. Model III: A Five-Period Planning Formulation

3. Conditions for Unconstrained Optimality

In order to determine the necessary conditions for unconstrained optimality, in the case of Model III, objective function (5.6) may be differentiated with respect to the decision variables, N_{ky}, k=1,...,M.[13] For a problem containing M NEC programs in each planning period, this differentiation yields 5M first order conditions of the form:

$$\text{(5.7)} \quad \frac{dD}{d(N_{ky})} = t_k P_{ry} \frac{S^o_{ry}}{R_{ry}} - \sum_{y=1}^{6-y} \sum_{k=1}^{m} \left[C^k_{1y} P_{ky} \frac{S^o_{ry}}{R_{ry}} \right] = 0$$

for y = 1,...,5 years and k = 1,...,M NEC programs.

The equations (5.7) may be solved to form the relationships

$$\text{(5.8)} \quad \frac{S^o_{k1}}{P_{k1}} = \left[t_k \frac{P_{r1}}{P_{k1}} \frac{S^o_{r1}}{R_{r1}} \right] - \left[t_k C^k_{12} \frac{P_{r2}}{P_{r1}} \frac{S^o_{r2}}{R_{r2}} \right] + \left[t_k [(C^k_{12})^2 - C_{12} C_{23}] \frac{P_{r3}}{P_{k1}} \frac{S^o_{r3}}{R_{r3}} \right]$$

$$+ \left[t_k [2(C^k_{12})^2 C^k_{23} - (C^k_{12})^3 - C^k_{12} C^k_{23} C^k_{34}] \frac{P_{r4}}{P_{k1}} \frac{S^o_{r4}}{R_{r4}} \right]$$

$$+ \left[t_k [(C^k_{12})^4 - 3(C^k_{12})^3 C^k_{23} + 2(C^k_{12})^2 C^k_{23} C^k_{34} + (C^k_{12})^2 (C^k_{23})^2 \right.$$

$$\left. - C^k_{12} C^k_{23} C^k_{34} C^k_{45}] \frac{P_{r5}}{P_{k1}} \frac{S^o_{r5}}{R_{r5}} \right]$$

for k=1,...M NEC programs.

---

[13]Again, the objective function of Model III is a convex function.

Equation (5.8) defines the dependencies, at unconstrained optimality, between the predicted manning levels for rating programs in various planning periods and the decision to meet NEC program goals in the first planning period. Equation (5.8) also reveals the subtle relationships which exist between the decision variables and the planning parameters of various years. Assume, for instance, that $C^k_{12} > 0$ for all NEC programs and that all other continuation rates are zero. In this case it can be shown, using equation (5.8), that propensity to fulfill first period NEC program goals still depends, in part, on the planned rating shortages for the third, fourth, and fifth year. This is true despite the fact that no trainee from the first planning year can possibly survive to fulfill NEC program requirements of the third and subsequent planning years. The indirect relationship between the first and third planning periods, in this case, results from the chain of dependency between the first and second periods and the second and third periods.

The conditions (5.8) provide a basis for several interesting observations concerning the effect of continuation rates on the optimal first period training strategy. <u>For the case in which continuation rates are constant for the k^{th} NEC program</u> ($C^k_{12}=C^k_{23}=C^k_{34}=C^k_{45}$), <u>the effect of the third and subsequent planning periods is always zero and the five-period planning problem reduces to the two-period structure</u>. For decreasing continuation rates ($C^k_{12} \geq \ldots \geq C^k_{45}$) and mild rating shortages (one to ten per cent) in the third and subsequent periods the net effect of the last three planning periods is to decrease slightly the propensity to meet first period NEC program goals. In general, a converse effect results from a sequence of increasing continuation rates.

Equation (5.8) demonstrates that the manner in which individuals, in a given NEC program, tend to survive from one year to the next can significantly affect the optimum first period NEC training strategy. The exact continuation rates within an NEC program may determine whether the propensity to meet first period goals should be increased, decreased, or remain unchanged as the result of a given projection for rating manning in future periods.

4. Use of Model III in an Assumed Planning Environment

Optimality conditions (5.8) are somewhat overwhelming and unintuitive in their general form. In an actual planning environment, however, numerical values for most of the planning parameters of (5.8) would be known, or could be estimated. In such a situation (5.8) could be utilized effectively to determine approximate adjustments to training solutions obtained on basis of a single period planning horizon.

In order to demonstrate the above, the author assumed continuation rates and a psychological discount rate for a sample NEC planning scenario as follows:

(1) Continuation rates for all NEC programs were assumed to be:

$C^k_{12} = .94$ $\qquad$ $C^k_{34} = .75$

$C^k_{23} = .80$ $\qquad$ $C^k_{45} = .35$

The above values were determined using a weighted average of total Navy enlisted continuation rates in the years 1959-1969. The author does not contend that the use of all Navy rates to approximate individual NEC program continuation rates is a particularly valid procedure. However, in the absence of historical data concerning the latter, it provided an appealing alternative.

(2) A psychological discount rate of 15% was assumed. This discount rate is, in the author's opinion, a rather conservative estimate. The present value factors associated with a 15% psychological discount rate can be calculated from (5.1) to be:

$$(PVF)_1 = 1.0$$
$$(PVF)_2 = .8696$$
$$(PVF)_3 = .7561$$
$$(PVF)_4 = .6575$$
$$(PVF)_5 = .5718$$

Using the continuation rates of assumption one above optimality condition (5.8) can be rewritten:

$$(5.9) \quad \frac{S^o_{k1}}{R_{k1}} = t_k \frac{P_{r1}}{P_{k1}} \left[\frac{S^o_{r1}}{R_{r1}} - .94(PVF)_2 \frac{S^o_{r2}}{R_{r2}} + .132(PVF)_3 \frac{S^o_{r3}}{R_{r3}} + .019(PVF)_4 \frac{S^o_{r4}}{R_{r4}} + .216 \frac{S^o_{r5}}{R_{r5}} \right]$$

Adjusting (5.9) for the assumed psychological discount rate of 15% yields the relation:

$$(5.10) \quad \frac{S^o_{k1}}{R_{r1}} = t_k \frac{P_{r1}}{P_{k1}} \left[\frac{S^o_{r1}}{R_{r1}} - .82 \frac{S^o_{r2}}{R_{r2}} + .10 \frac{S^o_{r3}}{R_{r3}} + .01 \frac{S^o_{r4}}{R_{r4}} + .12 \frac{S^o_{r5}}{R_{r5}} \right]$$

Assuming that goals weights have been determined utilizing methodology suggested in this paper, then P_{ri} is numeraire, and its value is one. Therefore (5.10) can be rewritten:

$$(5.11)\quad \frac{S^o_{k1}}{R_{r1}} = \left[\frac{t_k}{P_{k1}}\ \frac{S^o_{r1}}{R_{r1}}\right] + \underline{\frac{t_k}{P_{k1}}\left[-.82\ \frac{S^o_{r2}}{R_{r2}} + .10\ \frac{S^o_{r3}}{R_{r3}} + .01\ \frac{S^o_{r4}}{R_{r4}} + .12\ \frac{S^o_{r5}}{R_{r5}}\right]}$$

The first term on the right hand side of relation (5.11) is the condition for unconstrained optimality in the case of the single-period planning model. The remainder of the right hand side, which is underlined, represents the total effect, on first period training strategy, of extending the planning horizon four additional years.

For a given NEC program, the underlined portion of (5.11) is a function of down time due to training, first-period goal weight, and projected rating shortages in future years. Training down time and first period NEC program goal weights should be known if the information required to work a single period model has been properly gathered. Estimates of rating shortages in future planning periods may be obtained from external sources such as the ADSTAP.[14] Consequently, for each NEC program, the numerical value of the underlined portion of (5.11) may be estimated. If it appears to be significant (i.e. if it would change first year training input by one man or more) it may be utilized to make approximate adjustments to a computed single-period planning strategy.

5. <u>Use of Model III as a Computational Model</u>

Model III is formulated as a quadratic programming problem and could conceivably be solved using existing computer codes. However, due to the complexity of the formulation, both the computational effort and the problem of data input and output would be enormous relative to one of the single-period models.

---

[14] See Ref. 16 for a description of the ADSTAP System.

Aside from computational difficulty, there is the problem of variance in the value of the planning parameters over time. Model III treats the expected values of various planning parameters as certainty equivalents. The author has inferred that increased risk due to varying parameters would be accommodated by psychological discounting. It should be noted that there is a school of thought which disagrees with the use of discounting for these purposes (see Bierman [17], for instance). Whatever the method used to account for uncertainty in planning parameters, however, the greater risk associated with the multi-period case is detrimental to the optimization claims of the model.

In order to compute a meaningful numerical solution to Model III, it would be required that significant planning constraints be known up to five years in advance. In the author's opinion, such a lead time is not realistic. Often the lead times on important planning constraints, such as a budgetary limitation, are less than a year.

Another disadvantage to computational use of Model III is that the formulation requires data which, to the author's knowledge, does not exist. As demonstrated by (5.8) the multi-period training decision is sensitive to the continuation rates for individual NEC programs. It is foreseeable that, in specific situations, it would be appropriate to approximate NEC programs continuation rates with those of the source rating. Additional study is required to substantiate this procedure, however.

In view of the foregoing discussion, one might correctly infer that Model III is not proposed as a computational device to solve the multi-period Class C School planning problem. It was proposed primarily, as an analytical means to provide insight into some of the characteristics

and information requirements of the multi-period problem. At best, it might be utilized to make pragmatic adjustments to the training strategies produced by a single-period formulation.

VI. SUMMARY AND SUGGESTIONS FOR FURTHER STUDY

A. SUMMARY

Enlisted personnel planners in BUPERS are tasked with meeting quantitative and qualitative requirements on the Navy's enlisted manpower force. Within each Navy enlisted rating, various requirements compete for personnel resources. This paper has been concerned with the competing requirements of a rating program and the individual NEC programs within a rating. Currently, personnel allocation decisions in support of each of these requirements are made independently. In this paper, interdependence between these decisions has been studied.

The necessity to suboptimize in analyzing the problem has been pointed out. Major implications of viewing personnel allocation problems from a decentralized (BUPERS) point of view have been discussed.

Several analytical and computational models which consider collectively the decisions to meet overall rating requirements and NEC requirements within rating, have been proposed. These models utilized the concept of quadratic loss to simulate the goal-seeking behavior of BUPERS planners relative to a set of competing personnel goals. Analytical results indicated that there does exist dependence between personnel allocation decisions concerning NEC program requirements and overall rating requirements. Furthermore, this dependence is directly proportional to the percent shortage of rating personnel and the non-productive time associated with input to an NEC program. It is inversely proportional to the subjective priority of an NEC program. A method and sample questionnaire for determining the subjective priority of individual NEC programs has been presented.

The impact of projected rating shortages, in future years, on the decision to meet current NEC requirements has been investigated. It has been shown that the manner in which future personnel shortages should influence the decision to meet current NEC requirements is highly dependent on individual NEC program continuation rates. It should also depend on the time preferences or psychological discount rate of agencies responsible for NEC training decisions. A method of actually determining time preference has been proposed.

B. SUGGESTIONS FOR FURTHER STUDY

The Training Plan Goal Programming Models, presented in this paper, consider the NEC planning problem of each Navy rating separately. This assumes that planning dependencies or spillover effects between ratings is so small that they need not be considered. The veracity of this assumption should be confirmed by further study.

Agreement that the NEC planning problem may be considered independently, by rating, depends, in part, on one's views concerning the ability of BUPERS to control the relative manning shortages in enlisted ratings. Large personnel systems are known to have a great deal of inertia. Therefore, one might tend to accept the hypothesis that the manning shortage in a rating is fixed inside a small planning horizon. As the planning horizon is extended, however, it becomes more probable that allocation decisions regarding total enlisted assets can effectively control the relative manning shortages in Navy ratings. Therefore, for a sufficiently large planning horizon, in the NEC planning problem, it becomes appropriate to consider the shortages in ratings as decision variables, rather than as fixed exogenous inputs to the TPGP models.

It seems, to the author, that an extended version of the TPGP models could be utilized to investigate the significance of spillover effects between the NEC planning problems of various ratings. In such a formulation the planning problems of many ratings would be considered collectively. Within each rating the manning shortage over a planning period would be treated as a decision variable instead of a fixed input. It's quite likely that, under these conditions, the decisions to allocate total Navy personnel resources to various ratings would be somewhat sensitive to the NEC training loads of individual ratings. Revised rating shortages would in turn effect NEC program training decisions and ultimately the multi-rating NEC planning formulation could be utilized to determine whether or not significant benefits could be achieved by collective consideration of the NEC requirements of several ratings.

APPENDIX A

FORMULATION OF THE NEC PROGRAM GOALS THE NEC TRAINING RESOURCES MODEL

A. GENERAL

The proposed Training Plan Goal Programming (TPGP) models are multi-goal models. They accept, as exogenous inputs, a single operational goal for each NEC program. It is expected that the BUPERS NEC Training Resources (NEC-TR) model, described in Ref. 8, will be utilized to formulate the individual program goals for each NEC program. Because of the expected tie-in between the NEC-TR Model and the TPGP Model it was considered appropriate to briefly review methodology utilized by the former.

B. NEC-TR

1. Adjustment of Input Data

Figure A.1 is a flow diagram of the NEC-TR Model. Base NEC program requirements are exogenous inputs. Because it is expected that a certain percentage of the trained population will be in a non-productive status (transient, patient, or prisoner), or forced out of billet by sea/shore rotation, base requirements are adjusted using historically derived factors to obtain adjusted program requirements. Personnel inventories are obtained for the start of the planning period (fiscal year) and, using historical personnel loss factors, total expected personnel losses over the planning period are determined. Required school outputs are then computed on the basis:

> "For any (fiscal) year, the number of personnel which must be graduated from a particular course of instruction will be that number which will cause the end-of-the-year (NEC) personnel inventory and (NEC) requirements to be equal."

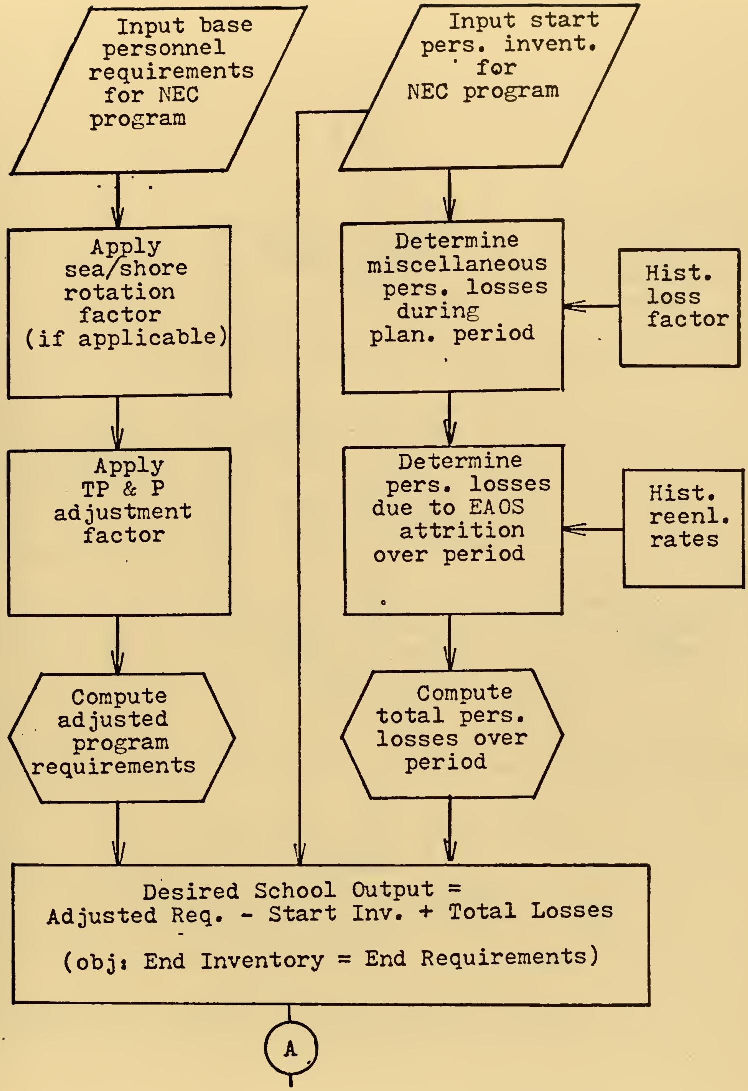


Figure A.1. Block diagram of NEC-TR Model

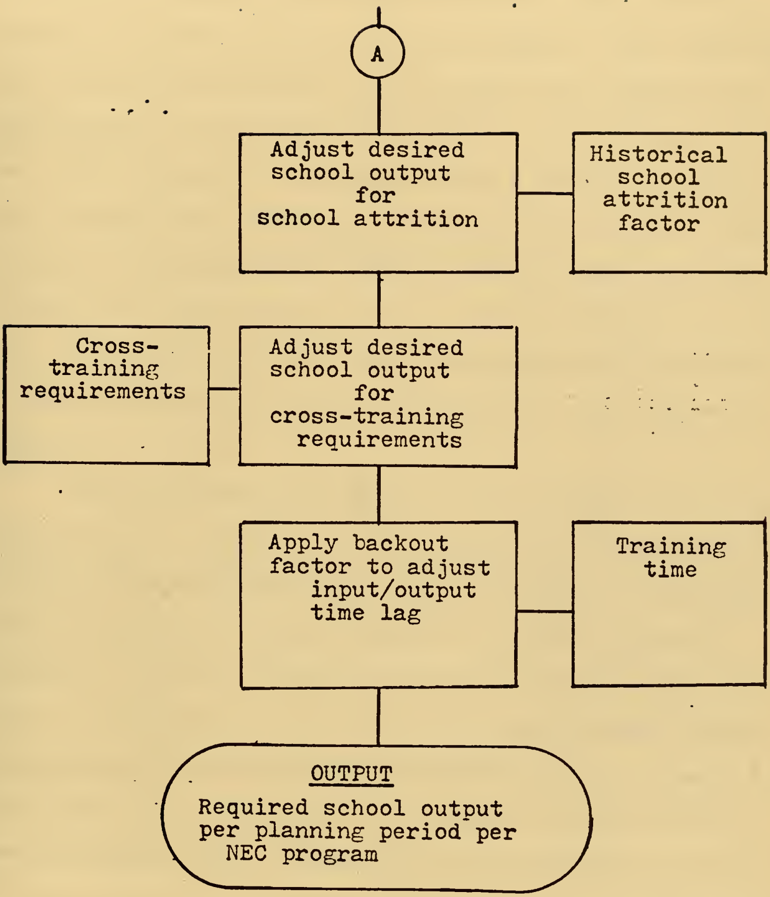


Figure A.1. Block diagram of NEC-TR model (continued)

2. Discussion of Main Planning Criterion

Preference of the above planning criterion to alternative criterion can be partially explained with the aid of two simplifying assumptions concerning the NEC planning environment. First, assume that once a school training quota is set for a fiscal year, school input, school output, and, consequently, NEC input is uniform throughout the year. This assumption is supportable on both a historical and logical basis. Steady, full, employment of training activity assets is a partial objective of training planning, itself. Secondly, assume that total losses, within an NEC program, occur at a constant rate over the planning year. This assumption is often used in personnel work and modelling. In this case, the author has not collected data to either support or reject it.

Assuming constant personnel loss rate and constant school output, Figure A.2 (a) is a graphical representation of the NEC training planning problem, with an initial shortage of personnel. The training planner desires to establish a planned inventory over the year (BE) which coincides as closely as possible with adjusted requirements (AD). The point F, in Figure A.2 (a), defines the time during the year at which planned personnel inventory equals adjusted requirements. As point F is varied between point A and point D all possible training plans which meet requirements at some time during the year are mapped out. Thus, the planner may vary F to find the training plan which best satisfies his criterion for minimizing the difference between personnel inventory and requirements. The difference between inventory and requirements over the year is representable as the areas of the triangles AFB and EFD in Figure A.2 (a).

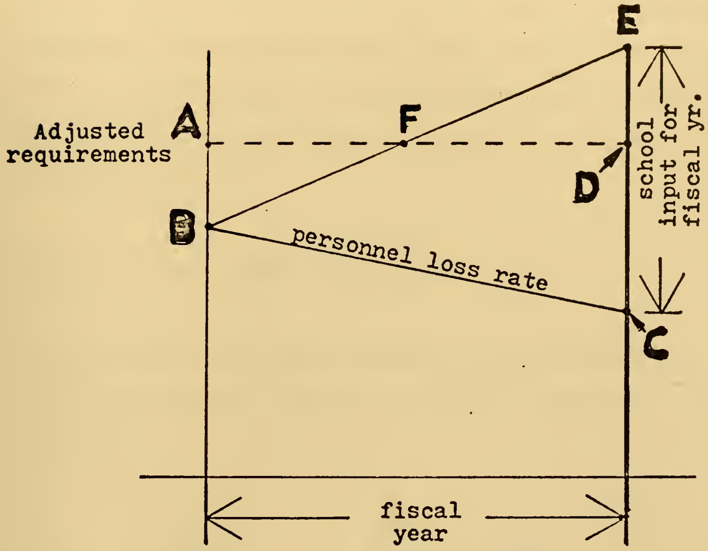


Figure A.2(a). A graphical representation of the NEC training planning problem.

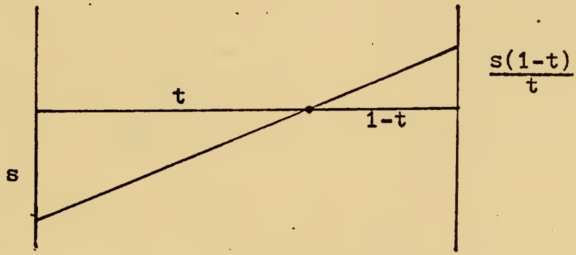


Figure A.2(b). A simplified version of Figure A.2(a).

Because of the necessary similarity between the triangles AFB and EFD in Figure A.2 (a) they can be redrawn, as in Figure A.2 (b). Here, S is the initial personnel shortage and t is the time from the beginning of the planning period at which inventory equals requirements. If t is varied from zero to one all training plans are mapped out. For any plan, the deviation between requirements and inventory over the planning period is:

$$\text{(a.1)} \qquad \frac{St}{2} + \frac{S(1 - t)^2}{2}$$

If the training planner has relative loss (dissatisfaction), L^+ and L^-, for trained personnel excesses and shortages, respectively, then a good plan would:

$$\text{(a.2)} \qquad \text{Minimize} \qquad L^- \frac{St}{2} + L^+ \frac{S(1 - t)^2}{2t}$$

By taking the derivative of (a.2) with respect to the variable t it can be shown that the t which minimizes (a.2) is:

$$\text{(a.3)} \qquad t = \sqrt{\frac{L+}{L^- + L^+}}$$

By (a.3), when $L^- = L^+$, t = .707.

Figure A.3 is a plot of the overall deviation defined by formula (a.1) for various values of t. Although deviation from requirements is at a minimum for t = .707 , it is relatively insensitive to variation of t in the range t = .5 to t = 1.0 . For all values of t, however, overall deviation from requirements remains directly proportional to, S,

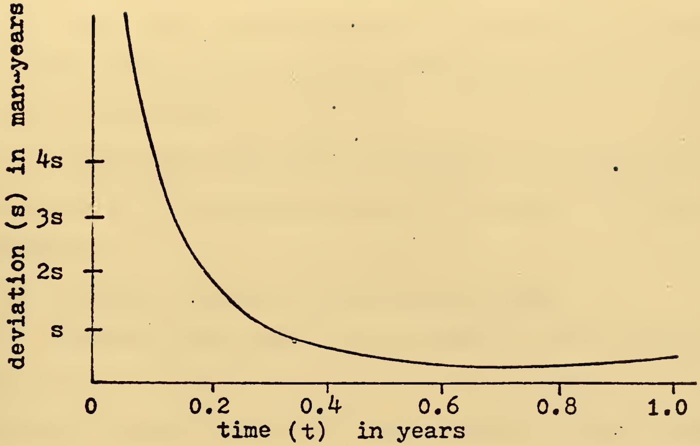


Figure A.3 Change in deviation as t is varied.

the initial personnel shortage at the beginning of the planning period. It can be seen from Figure A.2 (b) that although S is fixed for the current planning period the plan which has the best chance of resulting in a small S for the following planning period is the plan associated with $t = 1.0$. This is the plan obtained by the NEC-TR model criterion,

"end-of-the-year personnel inventory = adjusted requirements."

In retrospect then, the NEC-TR Model planning criterion appears to be a very reasonable long-range planning criterion subject to the conditions that:

(1) The assumption of uniform training rates and personnel losses over a planning period is valid for NEC programs.

(2) Weightings for shortages and excesses of NEC trained personnel are approximately equal.

(3) The planning methodology is not biased.

(In other words it produces actual end inventories which are distributed uniformly about adjusted requirements vice repetitive errors in one direction).

A development similar to the preceding can be carried out for the case of initial personnel excesses which results in identical conclusions.

3. <u>The Shift from School Output to School Input</u>

After the NEC-TR model has established the school output which will result in the required end inventory for each NEC the outputs are adjusted to account for expected school attrition and cross training. Cross training occurs in progressive NEC training programs where one program draws upon the resources of another.

At this point in the NEC-TR methodology, NEC program planning goals are in the form "required student output per Class C School per planning period." An important objective of training planning, however, is to provide fiscal guidance for the reprogramming of training assets to meet expected training demands. For this reason training plans, in particular Class C School Training Plans, are promulgated in terms of student inputs. In order to convert from student output/FY to student input/FY the NEC-TR Model applies a backout factor, to the output requirements of each planning period, which is proportional to the training time or course length requirement for each NEC program.

The final NEC-TR Model planning goal for each NEC program is of the form, "required input per NEC program per planning period." The TPGP Models accept single goals of this form for each NEC program in a given problem formulation.

APPENDIX B

A MATHEMATICAL CRITERION RESULTING IN A FAIR-SHARE DISTRIBUTION OF PERSONNEL TO PROGRAMS

The purpose of this appendix is to motivate, by example, the mathematical criterion utilized to determine preferred personnel distributions in the Training Plan Goal Programming formulations.

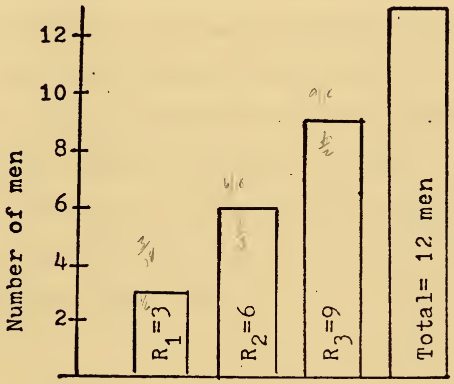


Figure B.1. A personnel distribution problem.

Consider the problem shown in Figure B.1 in which one has twelve men assigned to three programs of equal importance. The programs have requirements R_1, R_2, and R_3 of three, six, and nine men, respectively. Faced with a 33.3% overall shortage of personnel one desires to obtain a distribution of personnel to programs which minimizes the effect of the personnel shortage. Since it's desirable to solve future problems of this type on a computer, a mathematical formulation which yields the desired distribution of personnel is sought.

Sensing that the overall shortage must be a function of individual program shortage one might choose as his first criterion to make the total sum of the three program shortages as small as possible. If N_i is the number of men allocated to program i, then $(R_i - N_i)$ is the shortage in program i, and the objective is to:

$$\text{(b.1)} \qquad \text{Minimize} \quad \sum_{i=1}^{3} (R_i - N_i) \; .$$

But criterion (b.1) works rather poorly. It leads to no preferred personnel distribution. In fact, any arbitrary distribution of the twelve men to the three programs scores equally well. One difficulty associated with use of criterion (b.1) is that personnel excesses in one program may cancel personnel shortages in another.

In order to overcome this cancellation problem one might elect to choose another criterion; that of minimizing the sum of the absolute value of program shortages. That is:

$$\text{(b.2)} \qquad \text{Minimize} \quad \sum_{i=1}^{3} |R_i - N_i| \quad .$$

Criterion (b.2) does not permit overmanning of any personnel program when resources are scarce. However, any allocation of the personnel such that $N_i \leq R_i$ scores equally well under this criterion. It is possible, therefore, that (b.2) might prescribe a distribution of personnel which completely ignores program one or two while manning program three completely.

Since (b.2) does not necessarily result in a unique personnel distribution which recognizes the requirements of all programs one might try

the criterion; minimize the sum of the squared personnel shortages in each program:

$$\text{(b.3)} \qquad \text{Minimize} \sum_{i=1}^{3} (R_i - N_i)^2 \quad .$$

The personnel distribution which satisfies criterion (b.3) is unique. It is $N_1 = 1$, $N_2 = 4$, and $N_3 = 7$ or, in general, the allocation of personnel such that each program has exactly the same shortage. However, it is still possible to contrive examples in which a personnel distribution resulting from application of criterion (b.3) is unsatisfactory. For example, suppose one was distributing 990 personnel to two programs of size 10 and 1000. Since the programs would share the total personnel shortage equally all 990 men would be allocated to the larger program. Thus under criterion (b.3) it's possible for a program to receive greater than a fair-share of the personnel resources purely because it has larger personnel requirements.

Since from the beginning it has been assumed that all personnel programs are of equal priority, it's rather inappropriate to accept a distributional criterion which seem to relate program importance to program size. One might, therefore, correct the bias in (b.3) by normalizing each program for size. This yields the criterion:

$$\text{(b.4)} \qquad \text{Minimize} \sum_{i=1}^{3} \frac{(R_i - N_i)^2}{R_i} \quad .$$

In the sample problem, the distribution of available personnel uniquely satisfying criterion (b.4) is $N_1 = 2$, $N_2 = 4$, and $N_3 = 6$ or, in general, the allocation of personnel such that each program has the same

percent shortage (or same percent manning). No program, no matter how small, would be completely ignored as a result of personnel distributions resulting from this allocation process.

Finally, one may feel that he will face personnel allocation decisions in the future in which he is not willing to assume equal importance of all programs. Criterion (b.4) must, therefore, be revised to accommodate any subjective priorities one might have concerning a given group of programs competing for personnel resources. A final criterion might be:

$$\text{(b.5)} \qquad \text{Minimize} \sum_{i=1}^{3} \frac{P_i(R_i - N_i)^2}{R_i},$$

where P_i represents the subjective priority for the i^{th} program relative to other programs under consideration. If, for instance, P_1 is twice as large as other P_i's then program one will have only one-half the percent shortage of other programs.

Reviewing, the basic properties of criterion (b.5) are:

(1) If there is a positive amount of resource to be distributed no program is ignored.

(2) If all programs are of equal importance resources are fair-shared as a percent of program size.

(3) Since all deviations from program requirements are "positive" program excesses are as undesirable as program shortages

(4) It is possible to modify resource distributions when there are subjective priorities for the programs involved. When resources are scarce programs with higher weights will receive greater than a fair-share of the resources. The converse is true where there is an excess of resources.

Additionally, since formulations similar to (b.5) have been applied to a wide variety of problems, mathematical and computerized methods for solution of (b.5) exist and are readily available (see Ref. 11, for instance).

APPENDIX C

A METHOD FOR DETERMINING THE RELATIVE WEIGHT OF NEC PROGRAM GOALS

A. GENERAL

The purpose of this appendix is to propose methodology for determining the subjective goal weights for the various programs in the TPGP models.

B. A METHOD BASED ON QUADRATIC LOSS

One of the principle hypotheses of this paper is that BUPERS, as an organization, tends to exhibit quadratic loss for shortages in personnel programs. Under this hypothesis when all personnel programs are of equal priority, the preferred distribution of personnel to programs will be the one which exhibits identical manning percentages in each of the personnel programs. When the quadratic loss hypothesis holds, the converse is also true. If the preferred allocation of personnel to programs does not exhibit equal percent manning in each program then the priorities of the programs involved are not equal.[1] Furthermore, the exact manning percentages of the preferred allocation reveal, precisely, the unique relative weightings in the set of programs considered.

From the foregoing it follows that, a preferred allocation of personnel to programs reveals the relative weights between the programs. The

---

[1]It should be noted that a preferred distribution of personnel represents a desired state. It may or may not be reflective of the current manning in any of the personnel programs of concern.

problem of determining the relative weights for personnel programs is thus reduced to one of defining the preferred allocation of personnel resources for a given set of programs.

The following sample questionnaire is designed to assist training planners in ascertaining the preferred allocation for a set of programs. It is proposed as an example only and would probably require reworking prior to actual use.

SAMPLE QUESTIONNAIRE

This questionnaire is being submitted to you in an effort to ellicit better estimates of the priorities that we, in training planning, should associate with the personnel programs managed directly by you. As a rating manager, you have daily contact with activities that utilize personnel of the ________ rating. This provides you with an opportunity to sense directly the relative demands for personnel of the ________ rating which are qualified in various NEC categories. You are also uniquely aware of many of the long-range and short-run problems of the rating, in general. In view of these facts your judgements concerning the priorities for NEC programs within the ________ rating are particularly valued. The table below, is designed to assist you in expressing your views concerning these priorities. It contains a list of various NEC programs within the ________ rating for which BUPERS performs the associated Class C School planning. The first program listed is the rating, itself.

Column I of the table contains an estimate of the manning requirements of each personnel program listed. Assume that these requirements are fixed. Column II contains a fictitious initial inventory for each personnel program. In most cases you will find that the fictitious manning is less than 100%. Column III is labeled "Preferred Distribution." It is currently blank.

Your are to determine a preferred distribution by reassigning the distributable personnel in column two to the various progarms listed in the table until you have attained the assignment of personnel to programs which appears most satisfactory to you. Distributables may be moved between NEC programs and in or out of the rating. After you have determined the preferred distribution, recheck to ensure that the total of column II equals the total of column III. Finally, in column IV, enter the percent manning in each personnel program which resulted from your redistribution of the initial personnel inventories.

TABLE OF DATA FOR FISCAL YEAR __________

COL.	I	II	III	IV
PROGRAM	REQUIREMENTS	INITIAL INVENTORY	PREFERRED DISTRIBUTION	FINAL % MANNING
XX RATING	10,000	9,000		
NEC-1243	2,000	175		
NEC-2468	1,000	85		
....	.....	.....		

In the table of the sample questionnaire the initiating unit should fill in columns I and II. The figures in column I should approximate the actual personnel requirements of the programs listed. Requirements in the rating block should be total rating requirements and, therefore, should not be adjusted for the requirements of NEC programs appearing in the same list. The sample questionnaire emulates the view of the TPGP models, that NEC requirements are essentially qualitative requirements on a rating. That is, they are requirements to be met in addition to the normal rating requirements. In order to meet the NEC requirements one must pay an opportunity cost in terms of rating man-years. In the sample questionnaire the trade-off is on a one for one basis. A gain of one NEC man-year is achieved at an opportunity cost of one rating man-year. Since the TPGP models optimize on basis of the ratio P_k/t_k, the

models account for the fact that the opportunity costs of NEC programs actually vary and are generally less than one rating man-year.

The figures in column II essentially establish a resource constraint on the distribution problem. In the author's opinion, numbers in column II should be in the range 80-90% of program requirements. Actual experience should help to narrow the above range.

After the preferred distribution has been defined, the final percentages in column IV of the questionnarie may be utilized in conjunction with Table I.C to determine the relative weights for the programs listed.

In Table I.C percent rating shortages are measured along the vertical axis. Percent shortages in NEC programs are measured along the horizontal axis. The radial lines extending outward from the origin are lines of equal weighting factors.

To establish the weight for an NEC program listed in the sample questionnaire, one should locate the final percent shortage for the rating program on the vertical axis of Table I.C and draw a horizontal line. Similarly, one should draw a vertical line from a point of the horizontal axis which represents the percent shortage in the NEC programs. The intersection of these two lines defines a point. The relative weight for the NEC program may be obtained by interpolating between the radial weight lines which bound the point of intersection. All weight factors in Table I.C are based on a weight for the rating program of 1.0.

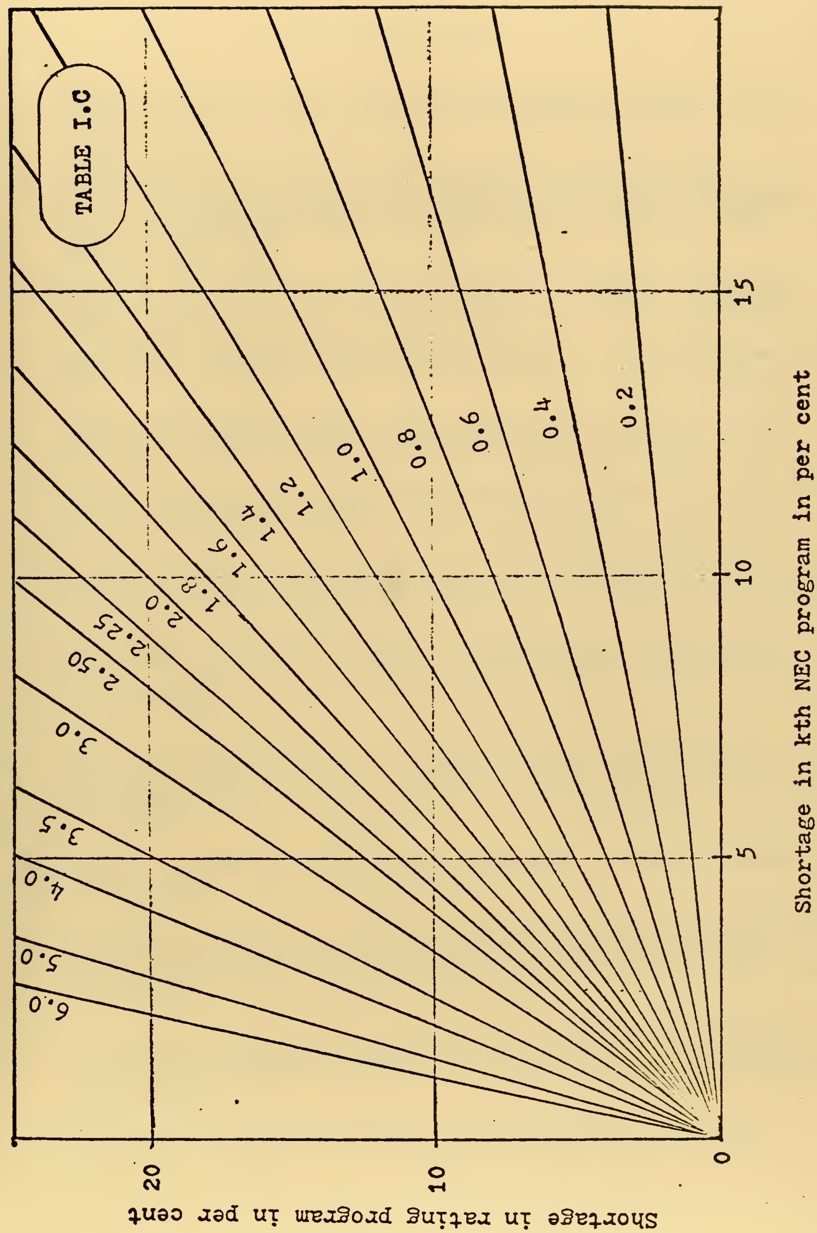


TABLE I.C
Shortage in kth NEC program in per cent
Shortage in rating program in per cent
0
5
10
15
20
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.25
2.50
3.0
3.5
4.0
5.0
6.0

LIST OF REFERENCES

1. Department of the Navy, Bureau of Naval Personnel Manual, NAVPERS 15701B, p. 00-1, October 1971.

2. Naval Personnel Research and Development Laboratory, Report WRM 70-26, The Navy Rating Structure and the Self Renewing Occupational Field (SROF) Concept: An Analysis, by Linsert, H. and others, March 1970.

3. Hitch, C., "Sub-optimization in Operations Research," Journal of the Operations Research Society of America, p. 87-99, May 1953.

4. DeGroot, M., Optimal Statistical Decisions, p. 227-231, McGraw-Hill Inc., 1970.

5. Ijiri, Y., Management Goals and Accounting for Control, p. 50-62, Rand-McNally & Co., 1965.

6. Johnsen, E., Studies in Multiobjective Decision Models, p. 400-409, Barnes & Noble Inc., 1968.

7. Department of the Navy, Chief of Naval Operations, OPNAVINST 1300.63, Subject: Enlisted Manning Level Priorities and Distribution Practices; Policy regarding, 21 July 1971.

8. Department of the Navy, Bureau of Naval Personnel, Memorandum, Subject: Procedures for Developing the NEC Plan; Promulgation of, 2 April 1969.

9. Charnes, A. and Stedry, A., "Investigations in the Theory of Multiple Budgeted Goals," p. 186-204 in Cooper, Leavitt, and Shelly, Management Controls: New Directions in Basic Research, McGraw Hill Inc., 1964.

10. Hadley, G., Nonlinear and Dynamic Programming, p. 90-93, Addison-Wesley, 1964.

11. RAND Corporation Report R-516-PR, A Computer Program for Quadratic Mathematical Models to be Used for Aircraft Design and Other Applications Involving Linear Constraints, by L. Cutler and D. Pass, June 1971.

12. International Business Machines (IBM) Corporation, Mathematical Programming System/360 (360S-CO-14X) Linear and Separable Programming User's Manual, 1970.

13. Department of the Navy, Bureau of Naval Personnel, Guide for Manpower Management, NAVPERS 94546, p. 20-22, 1971.

14. Groose, R. and Proshan, A., "The Annual Cycle: Planning-Programming-Budgeting," p. 23-41 in Enke, Defense Management, Prentice-Hall Inc., 1967.

15. Wagner, H., Principles of Operations Research, p. 273-275, Prentice-Hall Inc., 1969.

16 Naval Personnel and Training Research Laboratory, Research Report SRR-71-28, New Concepts in Enlisted Personnel Planning: Introduction to the ADSTAP System, Silverman, J., May 1971.

17. Bierman, H. and Smidt, S., The Capital Budgeting Decision, MacMillan Co., 1966.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Chief of Naval Personnel Pers-11b Department of the Navy Washington, D. C. 20370	1
3.	Naval Postgraduate School Department of Operations Research and Administrative Sciences Monterey, California 93940	1
4.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
5.	Assoc. Professor C. R. Jones, Code 55Js Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	1
6.	Prof. C. A. Peterson, Code 55Pe Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
7.	CPT. Arthur L. West c/o MG K. W. Collins 9 Palmetto Place Hilton Head Island, South Carolina 29928	1
8.	LCDR R. L. E. Prath c/o Dr. H. O. Copher 521 Churchill Drive Charleston, West Virginia 25301	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A Goal Programming Approach to Class C School Planning

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; March 1972

5. AUTHOR(S) (First name, middle initial, last name)

Robert Lee Prath

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1972	82	17

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Enlisted personnel planners in the Bureau of Naval Personnel are tasked with meeting quantitative and qualitative requirements in the Navy's enlisted manpower force. In many cases fulfillment of qualitative requirements involves the allocation of personnel to training programs. Several analytical and computational models are proposed which allow planners to determine the levels at which various competing personnel requirements should be met. The levels prescribed for programs involving training may be used to formulate training plans. The models utilize quadratic programming techniques.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Manpower Model						
Military Training						
Goal Programming						

DD FORM 1 NOV 65 1473 (BACK)
S/N 0101-807-6821

Thesis
P825 Prath 134493
c.1
A goal programming
approach to class C
School planning.